Predicting Chaotic Systems with Quantum Echo-state Networks

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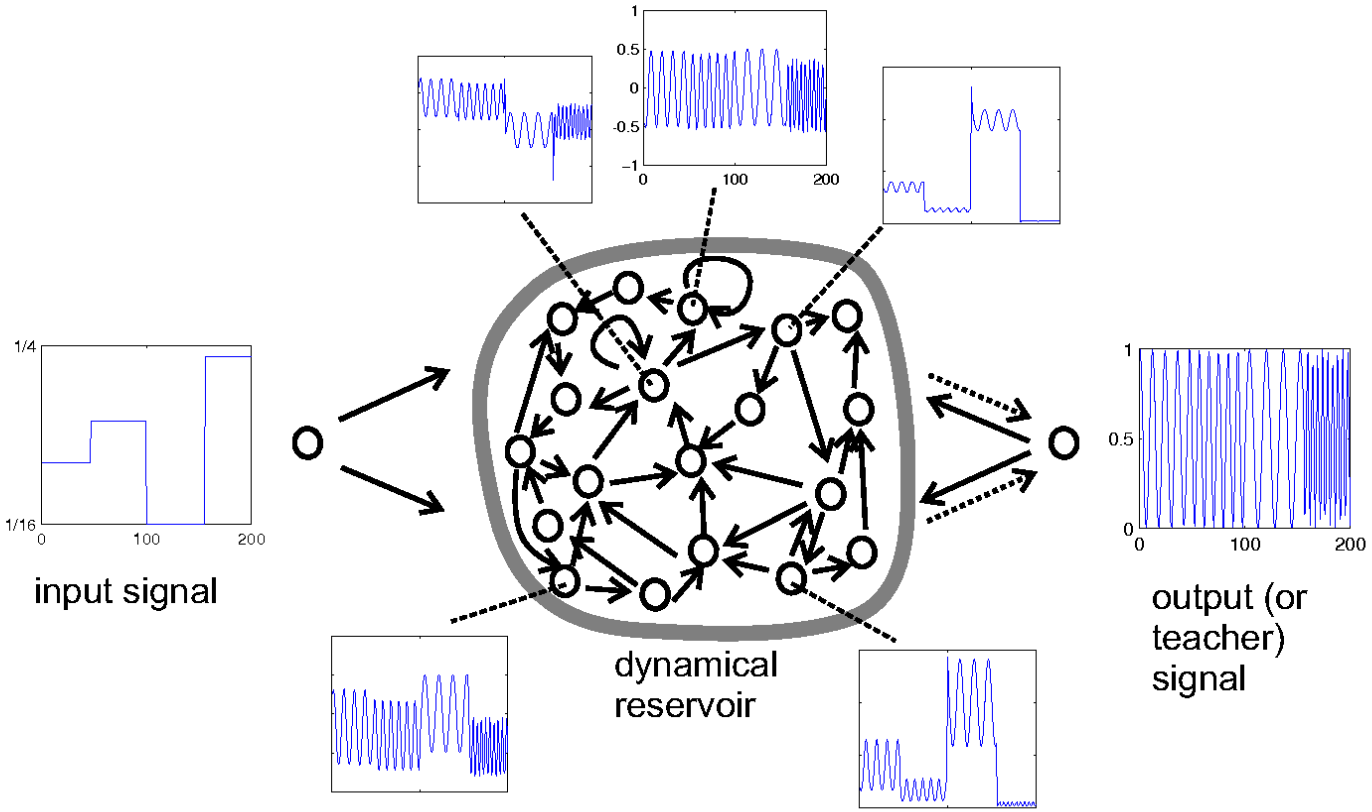
Quantum Echo-state Networks (QESNs)



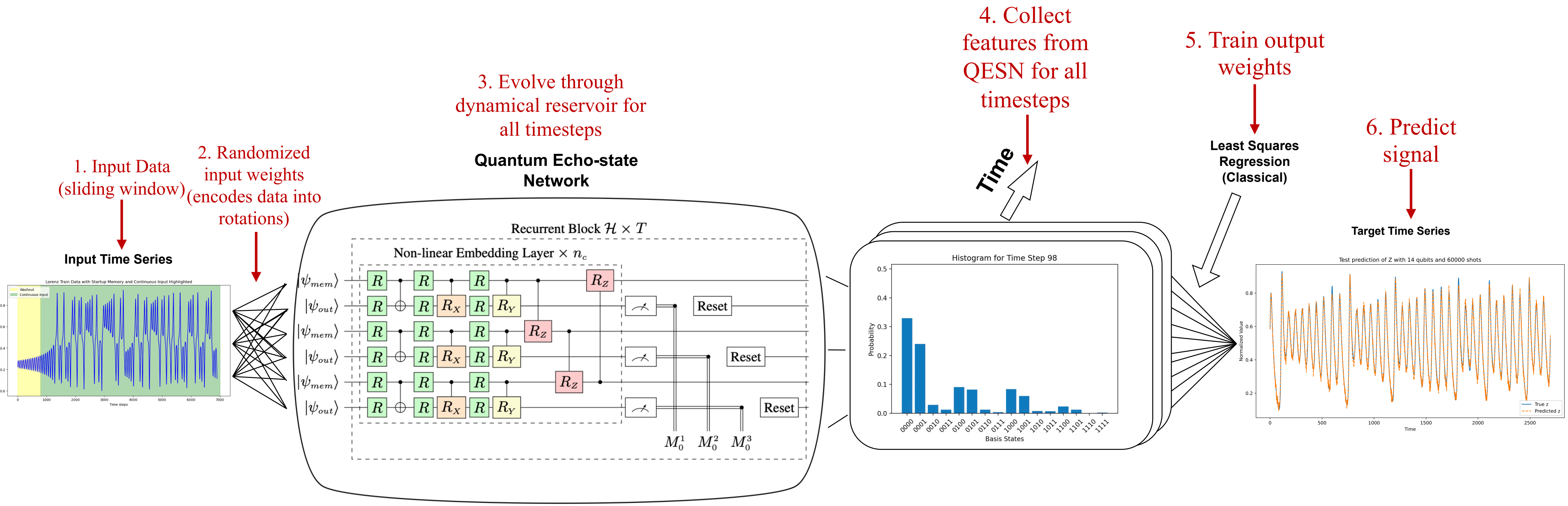
- Classical ESNs are lightweight sparsely allocated RNNs that are used for predicting dynamical systems.
- QESNs are their quantum counterpart and operate using qubits instead of “neurons”.
- Reservoir networks are used in time-series prediction, classification, and the predictions of chaotic PDEs and ODEs.
- Importantly, QESNs must also have **memory** and **nonlinearity**, which are both intrinsic properties of all RNNs.
- Implementing this sort of architecture on hardware for **long time-series** prediction has not been demonstrated due to noise and limitations of noisy-intermediate scale quantum (NISQ) coherence times.

Image: https://en.wikipedia.org/wiki/Echo_state_network

Main Contributions of the Paper:

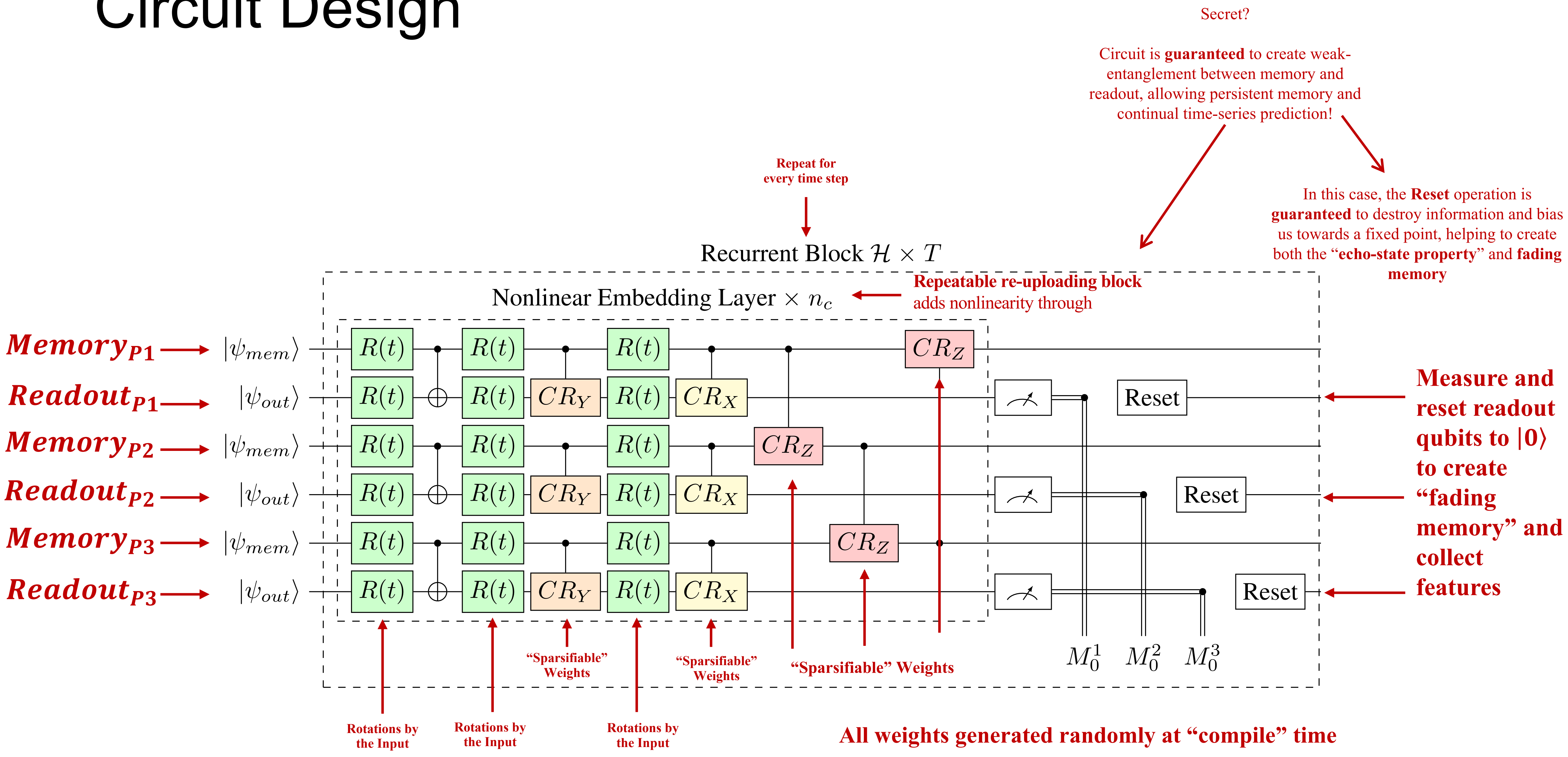
1. We introduce a scalable **QESN** algorithm that implements the two necessary components of **memory** and **nonlinearity** for quantum recurrent neural networks (QRNNs) on a quantum computer, which we verify with empirical analysis of simple input signals in numerical simulation.
2. We introduce and benchmark tunable hyperparameters such as **sparsity** and **repeatable data re-uploading blocks** which allow for more efficient circuits and tunable nonlinearity.
3. We demonstrate the capability of our QESN to accurately predict the chaotic Lorenz System set of ODEs using limited training information in numerical simulations.
4. We implement this design on IBM noisy-intermediate scale quantum (NISQ) hardware and conduct the first ever proof-of-concept for continuous long time-series prediction on IBM gate-based quantum computers with a circuit that ran 100 times longer than the median $\tau 1\ and\ \tau 2$ time of the IBM Marrakesh QPU.

QESN Pipeline & Algorithm



Features accumulated over 60,000 shots

Circuit Design



$$R_q(t) \leftarrow R_q(\alpha_t, \beta_t, \gamma_t) \leftarrow (\alpha_t, \beta_t, \gamma_t) \leftarrow \sum_{i=1}^{3} Win_i \cdot X_t + Wbias_i$$, where R(t) *is a unitary operator and* $\alpha, \beta, \gamma \in [0, \pi]$ *are Euler angles*

Response Analysis: Sparsity & Memory

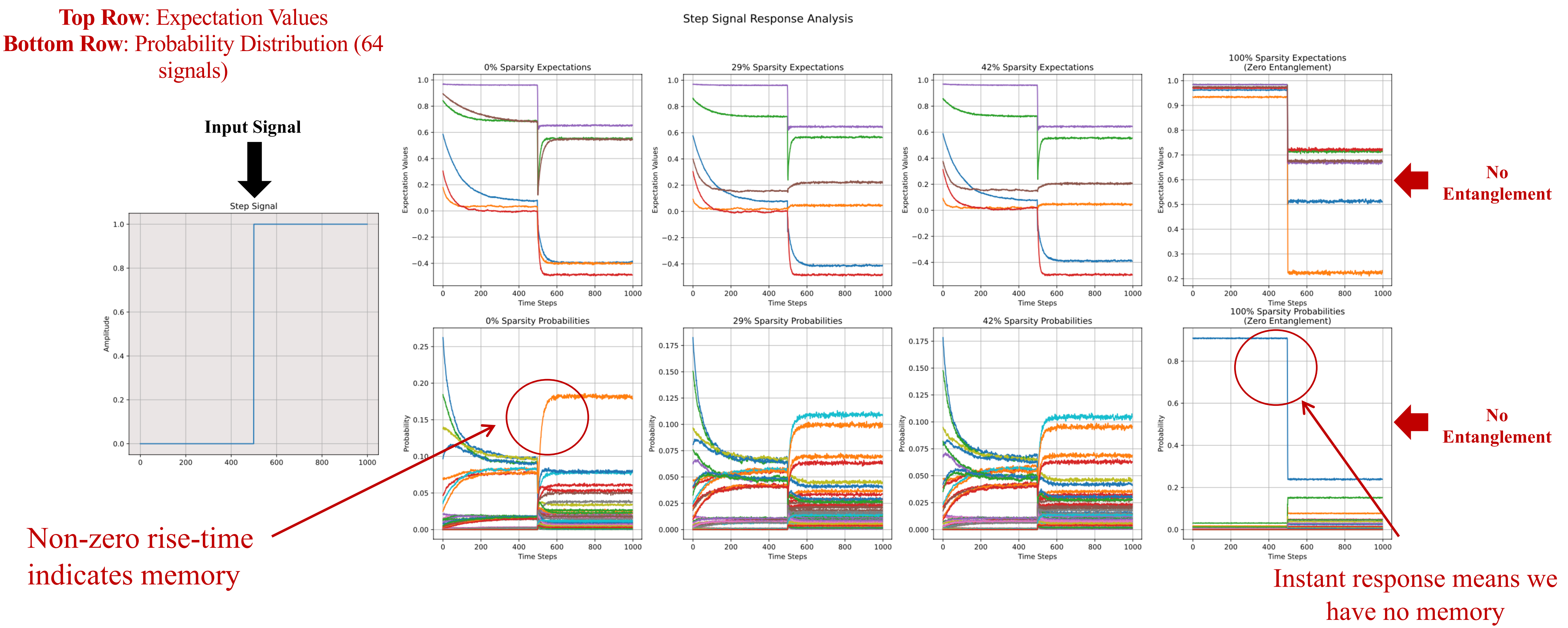
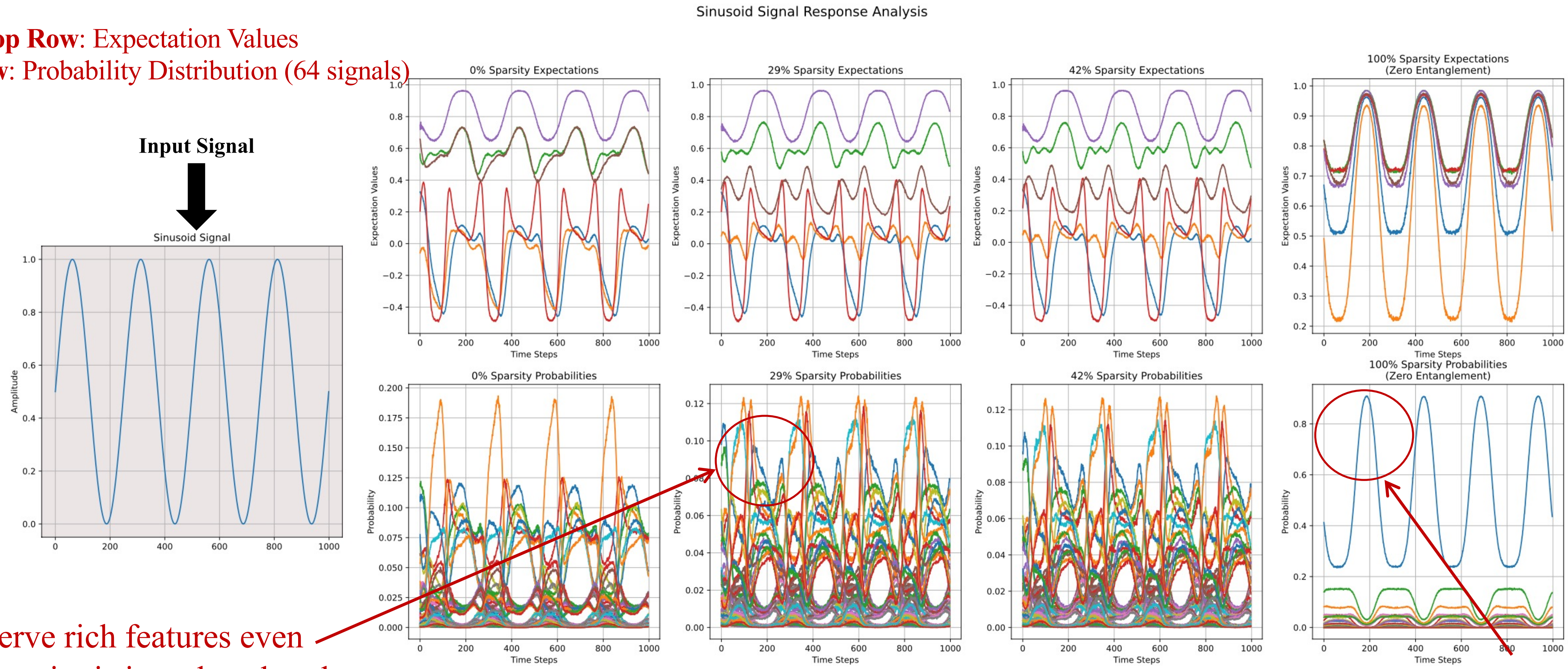


Figure 5: Step signal processing by the QESN circuit, with a focus on the rise-time and memory introduced by varying the entanglement and sparsity configurations.

Response Analysis: Sparsity & Expressivity

Top Row: Expectation Values
Bottom Row: Probability Distribution (64 signals)

Input Signal



No Entanglement

No Entanglement

We observe rich features even when sparsity is introduced to the circuit!

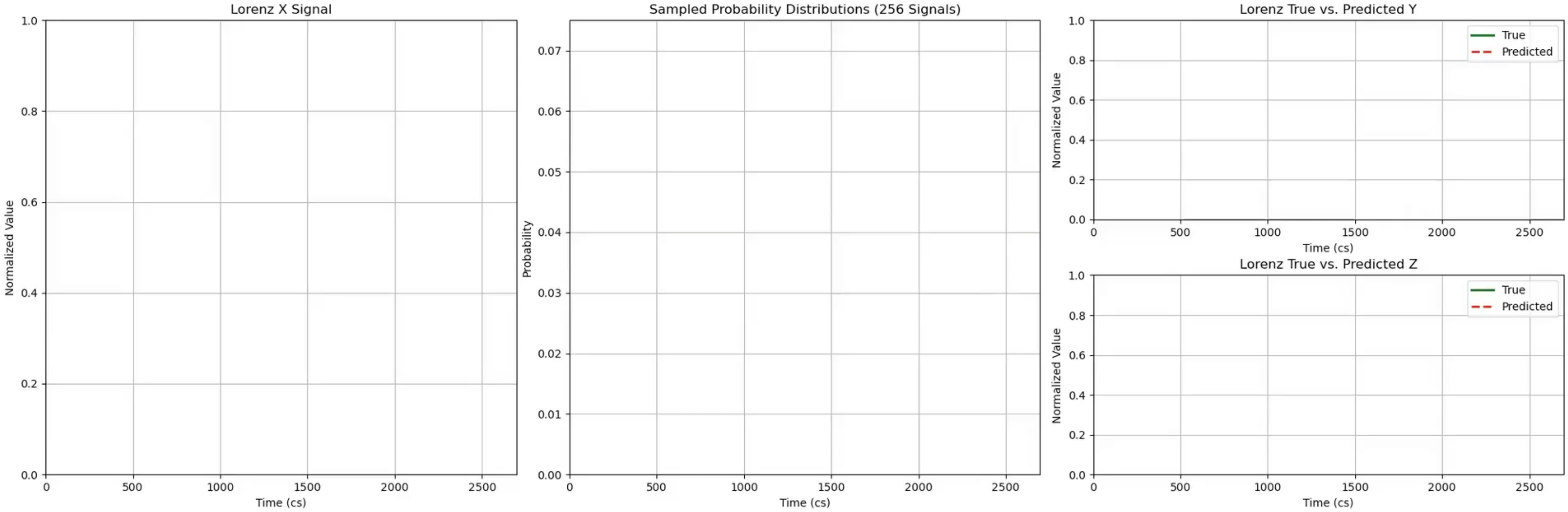
Simplistic features when entanglement is not present!

Figure 4: Response of the QESN circuit to a sinusoid input signal across different configurations, highlighting the richness of features with varying levels of sparsity and entanglement.

Sparsity means fewer gates, lower depth, and less errors

Fewer Gates

Results: Lorenz System (16 Qubits) (Test Set)



Aer Simulator **Train**: 6900 data points **Test**: 3000 data points **Washout**: 300 **Shots:** 60,000

Repeated Blocks: 3 **Sparsity:** ~50% **RMSE (test):** .0237

Results: Lorenz System (Simulator)

Qubits	Expectation Value	Probability Distribution	Distribution w. Noise
4 Qubits			
Train RMSE	**.1124**	.1177	.1468
Test RMSE	**.1112**	.1185	.2016
6 Qubits			
Train RMSE	.0986	**.0616**	.1193
Test RMSE	.0963	**.064**	.1315
8 Qubits			
Training RMSE	.0822	**.0429**	.1110
Test RMSE	.0798	**.0463**	.1285
10 Qubits			
Train RMSE	.0688	**.0425**	.1258
Test RMSE	.0699	**.0422**	.1298
12 Qubits			
Train RMSE	.0631	**.0378**	.0986
Test RMSE	.0635	**.0377**	.1282
14 Qubits			
Train RMSE	.0476	**.024**	.0754
Test RMSE	.046	**.0249**	.0988
16 Qubits			
Train RMSE	.0488	**.0225**	.0573
Test RMSE	.0493	**.0237**	.0895

~4 days to compute on DGX-A100 with all GPUs

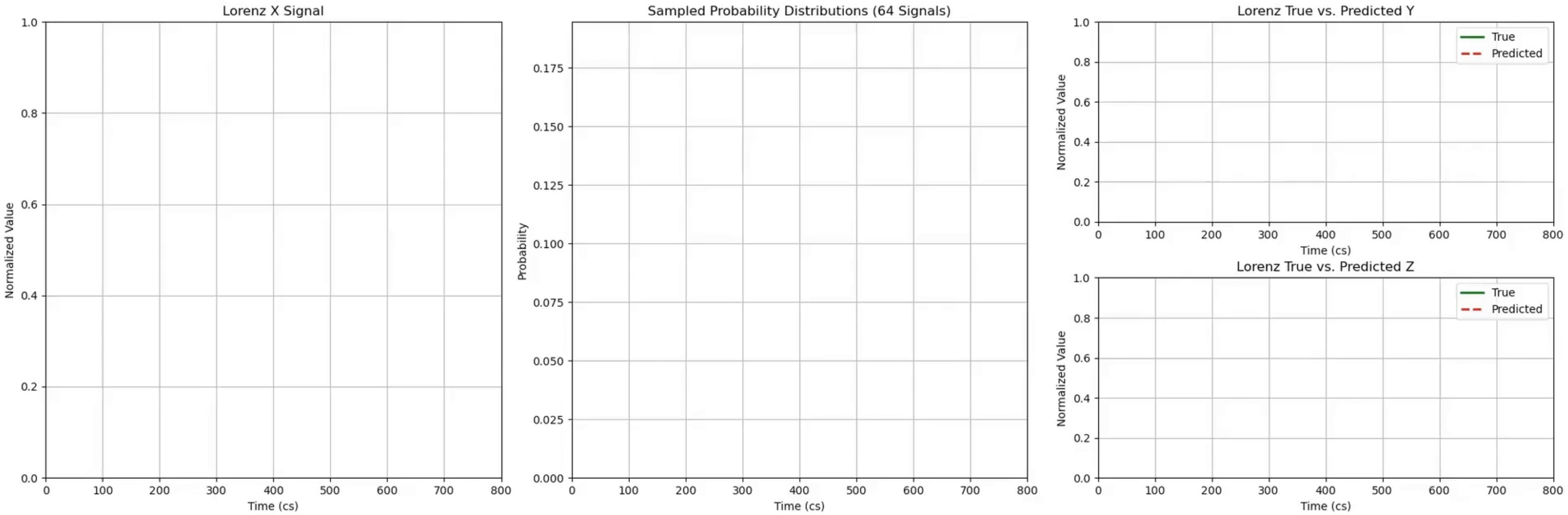
Table 7: Simulated training and test error using various different feature recovery methods and noise configurations measured in RMSE (Root mean squared error). An **IBM Fez** noise model was used to gather the noisy results. The best run from each category was used, and the elastic net regularization parameters were tuned for each bin to get lower test loss.

IBM Quantum Implementation

- Implementing quantum circuits is generally very difficult due to the short coherence times of qubits and intrinsic errors present in quantum computers.
- Our circuit design implements a "measure-and-reset" paradigm which allows for the creation of persistent memory in QCs and the ability to run a circuit indefinitely without intermediate halts [1][2][3]
- This hasn't been empirically validated on hardware because of many issues with excess measurement data on the IBM backend and noise.
- We demonstrate that our circuit can run for a timespan of ~48,000 μs and **successfully** predict the Lorenz System. This is due to the **guaranteed** weak entanglement the QESN circuit creates, as well as the "measure-and-reset" paradigm.
- We collect feature signals for a time length of 2000 data points on IBM hardware, greatly exceeding the previous bests of ~30 data points in circuits that only ran for ~200 μs [1]

[1] Hu, F., Khan, S.A., Bronn, N.T., Angelatos, G., Rowlands, G.E., Ribeill, G.J., Türeci, H.E.: Overcoming the coherence timebarrier in quantum machine learning on temporal data. Nature Communications 15(1), 7491 (2024)
[2] Yasuda, T., Suzuki, Y., Kubota, T., Nakajima, K., Gao, Q., Zhang, W., Shimono, S., Nurdin, H.I., Yamamoto, N.: Quantum reservoir computing with repeated measurements on superconducting devices (2023)
[3] Chen, J., Nurdin, H.I., Yamamoto, N.: Temporal information processing on noisy quantum computers. Phys. Rev. Appl. 14, 024065 (2020)

Results: IBM Marrakesh QPU (12 Qubits) (Test Set)



IBM QPU **Train**: 1200 data points **Test**: 800 data points **Washout**: 15 **Shots:** 60,000 **Repeated Blocks:** 3

Circuit execution time per shot > 48,000 μs **Median** $\boldsymbol{\tau_1} = 213.92\ \mu s$ & $\boldsymbol{\tau_2} = 119.57\ \mu s$

Sparsity: ~50% **RMSE (test):** .0922

Over **100 times longer** than the median τ_1 and τ_2 time!

Conclusion

- We designed and tested a Quantum Echo-state network (**QESN**) and proved empirically its capabilities at long-time series prediction using the chaotic Lorenz system.
- We show that our circuit has the necessary properties of **memory** and **nonlinearity,** two important components of classical RNNs that warranted further investigation in quantum circuits.
- We introduce tunable hyperparameters such as **sparsity** and **repeatable data re-uploading blocks** which allow for reduced circuit depth without sacrificing performance or output feature "richness", and controllable amounts of nonlinearity, respectively.
- We ran the circuit on IBM hardware demonstrating the first ever gate-based hardware validation of the "measure-and-reset" paradigm successfully executing for long-time series prediction with an experiment that ran over $\boldsymbol{100x}$ **longer than the median** $\boldsymbol{\tau_1 = 213.92\ \mu s\ \ and\ \ \tau_2 = 119.57\ \mu s}$ of the IBM Marrakesh QPU maintaining coherence and memory for the entire 2000 data point train and test set of the Lorenz System.

Questions?

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